On Belief Function Corrections

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(with discussions with Frédéric Pichon)

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Problem 1 The quality of the source may come in many guises.
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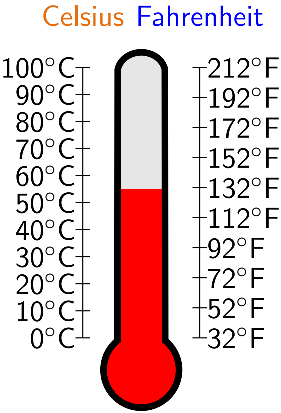
Problem 2 The quality of the source may only be known with some uncertainty.

- You want to know the temperature t.
- You take a thermometer T, which gives you a temperature of $t = 55°C$.



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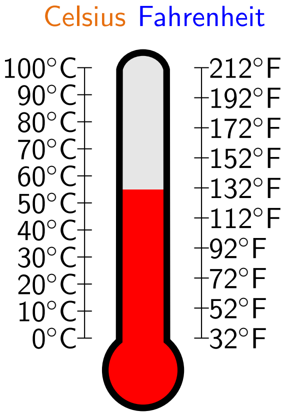

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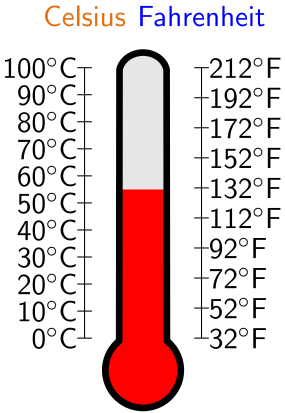

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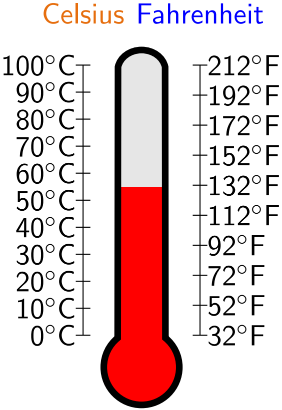

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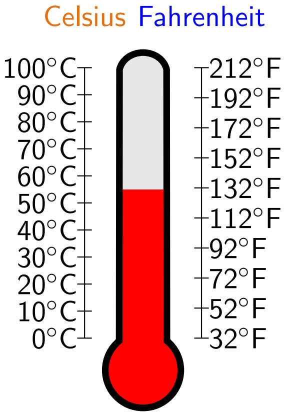

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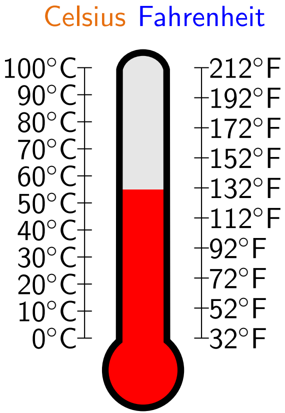

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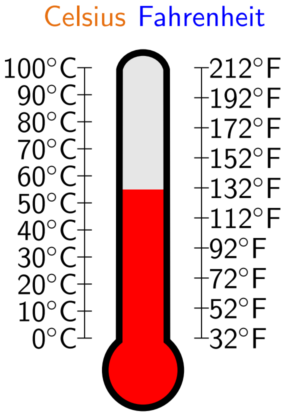

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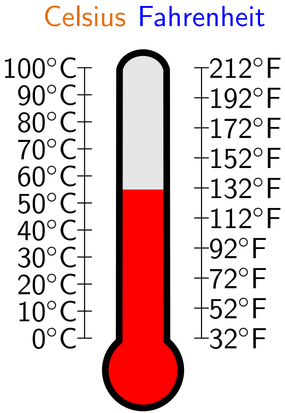

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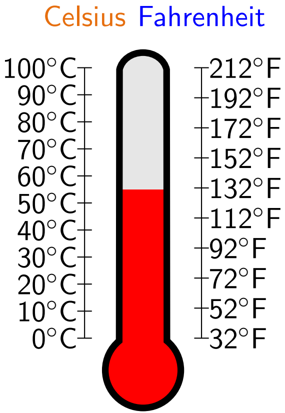

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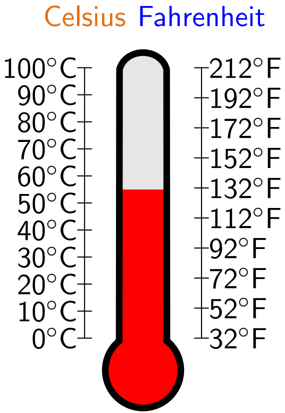

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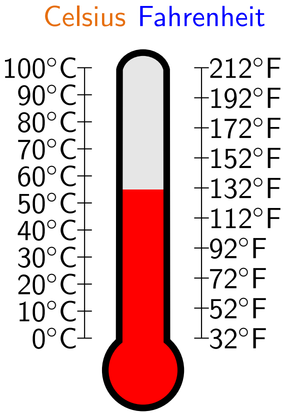

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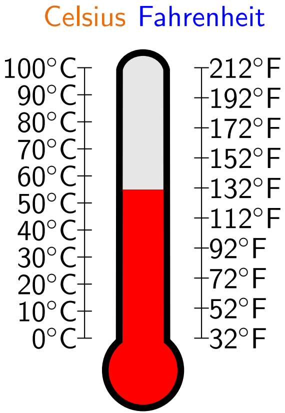
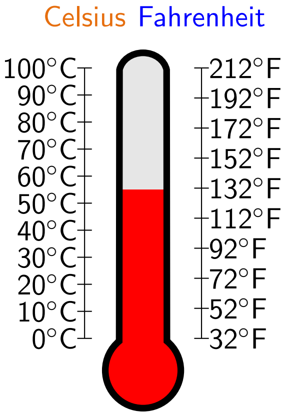
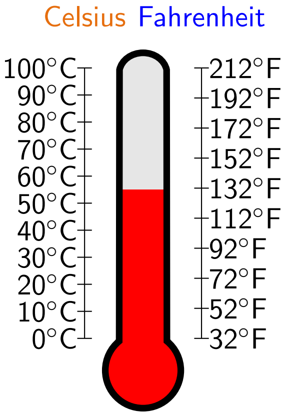


- The information on the quality of the source may also be uncertain and imprecise.



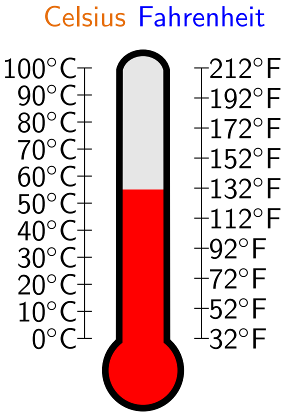
- The information on the quality of the source may also be uncertain and imprecise.
  - Example 6: One may believe to some degree that T is reliable.



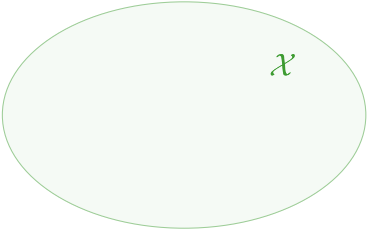
Introduction

The quality of the source may only be known with some uncertainty

- The information on the quality of the source may also be uncertain and imprecise.
  - Example 6: One may believe to some degree that T is reliable.
- Use of the belief function theory
  1. To model the information provided by the source.
  2. To model the information on the quality of the source.
  3. To infer the correction/adjustment of the information provided by the source according to the information on its quality.



Let **x** be a variable taking its values in a finite set $\mathcal{X}$.
You want to know its value x.



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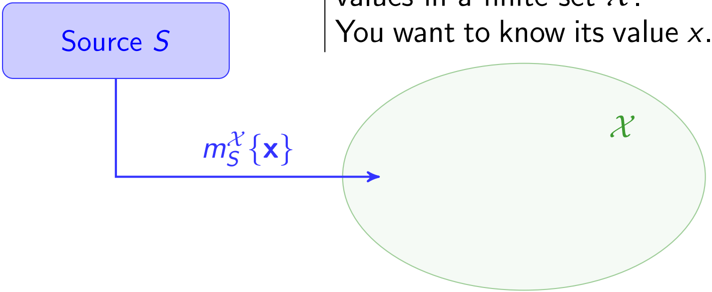

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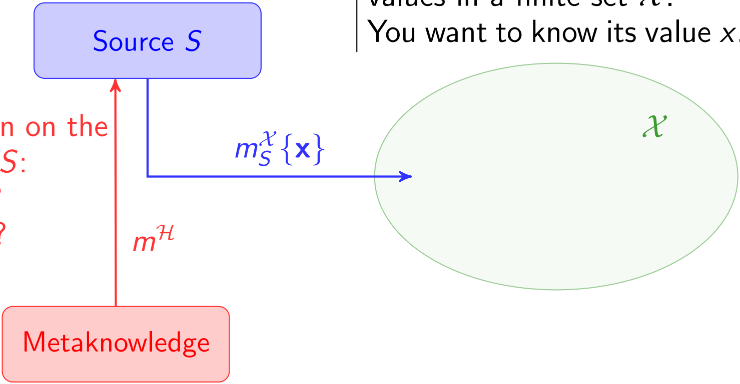

Information on the quality of S:
- Reliable?
- Truthful?
- Biased?

...

$\mathcal{H}$ set of possible states of S

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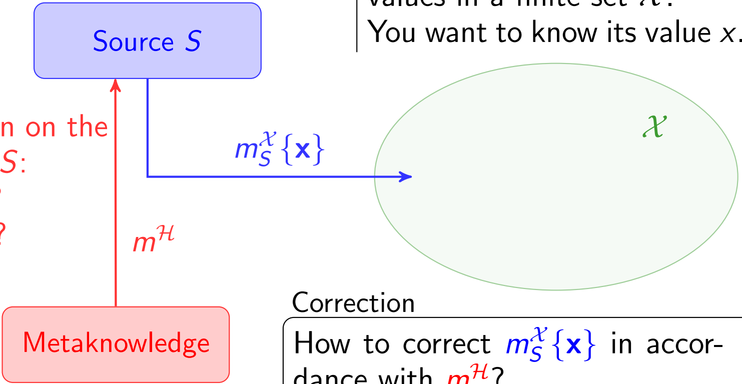

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Correction

How to correct $m_S^{\mathcal{X}}\{\mathbf{x}\}$ in accordance with $m^{\mathcal{H}}$?

1. Give an overview of correction models with their justifications / derivations.

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2. Show how to automatically learn some of them from labelled data (which can also help to build belief functions).

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2. Show how to automatically learn some of them from labelled data (which can also help to build belief functions).
3. Give examples/illustrations of the flexibility and expressivity power of the belief function theory.

Outline

Discounting

Contextual discounting based on a coarsening

Behaviour Based Correction (BBC)

Contextual discounting (CD), reinforcement (CR) and negating (CN)

Learning CD, CR and CN from labelled data

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Discounting of a mass function (MF) m is defined by (Shafer,1976):

$$\begin{cases} {}^{\alpha}m(A) & = & (1-\alpha)m(A), \quad \forall A \subset \mathcal{X}, \\ {}^{\alpha}m(\mathcal{X}) & = & (1-\alpha)m(\mathcal{X}) + \alpha \ , \end{cases}$$

where $\alpha \in [0, 1]$ is the discount rate.

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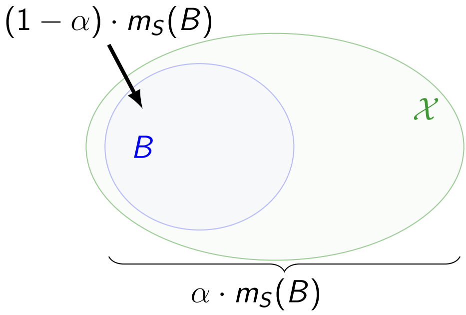
Example:

- $\mathcal{X} = \{x_1, x_2, x_3\}$
- $m(\{x_1\}) = .2$, $m(\{x_2\}) = .4$ and $m(\{x_1, x_2\}) = .4$
- With discount rate $\alpha = .2$:

$$\begin{cases} {}^{\alpha}m(\{x_1\}) & = & .8 \times .2 & = & .16 \\ {}^{\alpha}m(\{x_2\}) & = & .8 \times .4 & = & .32 \\ {}^{\alpha}m(\{x_1, x_2\}) & = & .8 \times .4 & = & .32 \\ {}^{\alpha}m(\mathcal{X}) & = & .8 \times .0 + .2 & = & .20 \end{cases}$$

For each focal element B of m_S:



- A part $(1-\alpha)\cdot m_S(B)$ remains on B.
- A part $\alpha \cdot m_S(B)$ is transferred to $\mathcal{X}$.

Discounting

Matrix representation (Smets, 2002)

Discounting ${}^{\alpha}m$ is a generalization of m (${}^{\alpha}m \sqsupseteq_s m$):

$$ {}^{\alpha}m(A) = \sum_{B \subseteq \mathcal{X}} {}^{\alpha}G(A,B)m(B) \ , $$

with ${}^{\alpha}\mathbf{G}$ a generalisation matrix defined by:

$$ {}^{\alpha}G(A,B) = \begin{cases} 1-\alpha & \text{if } A = B \neq \mathcal{X}, \\ \alpha & \text{if } A = \mathcal{X} \text{ and } B \subset A, \\ 1 & \text{if } A = B = \mathcal{X} \\ 0 & \text{otherwise.} \end{cases} $$

$$ {}^{\alpha}\mathbf{G} = \begin{pmatrix} 1-\alpha & 0 & \dots & 0 & 0 \\ 0 & 1-\alpha & \dots & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \dots & 1-\alpha & 0 \\ \alpha & \alpha & \dots & \alpha & 1 \end{pmatrix} $$

Discounting

Matrix representation: example

With $\alpha = .2$, $\beta = 1 - \alpha = .8$ and $\mathcal{X} = \{x_1, x_2, x_3\}$:

$$
\begin{array}{rl}
 & {}^{\alpha}m \quad = \quad {}^{\alpha}G(A,B) \quad \cdot \quad m \\
\begin{array}{l}
000 : \emptyset \\
001 : \{x_1\} \\
010 : \{x_2\} \\
011 : \{x_1, x_2\} \\
100 : \{x_3\} \\
101 : \{x_1, x_3\} \\
110 : \{x_2, x_3\} \\
111 : \{x_1, x_2, x_3\}
\end{array}
&
\begin{pmatrix} .0 \\ .16 \\ .32 \\ .32 \\ .0 \\ .0 \\ .0 \\ .20 \end{pmatrix}
=
\begin{pmatrix}
\beta & & & & & & & \\
 & \beta & & & & & & \\
 & & \beta & & & & & \\
 & & & \beta & & & & \\
 & & & & \beta & & & \\
 & & & & & \beta & & \\
 & & & & & & \beta & \\
\alpha & \alpha & \alpha & \alpha & \alpha & \alpha & \alpha & 1
\end{pmatrix}
\cdot
\begin{pmatrix} .0 \\ .2 \\ .4 \\ .4 \\ .0 \\ .0 \\ .0 \\ .0 \end{pmatrix}
\end{array}
$$

Discounting

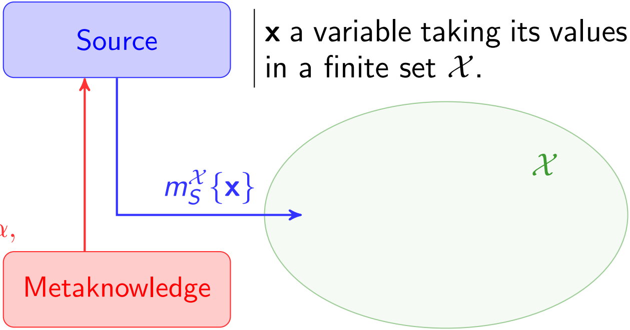
Derivation (Smets, 1993)

The source is reliable (r) or not ($\neg r$). $\mathcal{H} = \{r, \neg r\}$.

$m^{\mathcal{X}}[\{r\}] = m_S^{\mathcal{X}}$.

$m^{\mathcal{X}}[\{\neg r\}] = m_{\mathcal{X}}$.

$$\begin{cases} m^{\mathcal{H}}(\{r\}) = 1 - \alpha, \\ m^{\mathcal{H}}(\mathcal{H}) = \alpha \ . \end{cases}$$



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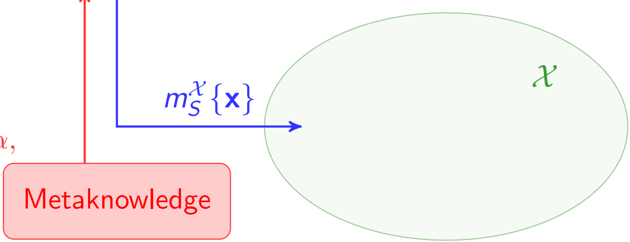


Discounting

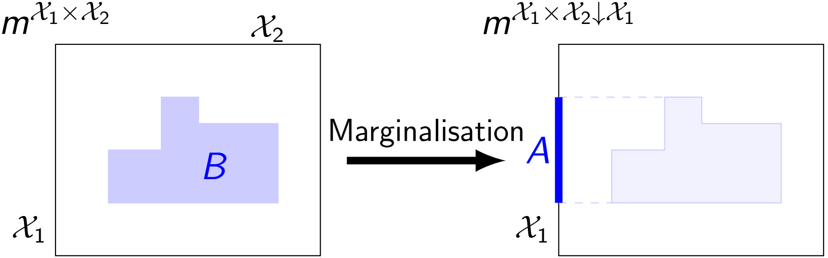
${}^{\alpha}m$ is then obtained from $m^{\mathcal{H}}$ and $m_S^{\mathcal{X}}$: ${}^{\alpha}m = \left(m^{\mathcal{X}}[\{\neg r\}]^{\Uparrow \mathcal{X}\times\mathcal{H}} \textcircled{\cap} m^{\mathcal{X}}[\{r\}]^{\Uparrow \mathcal{X}\times\mathcal{H}} \textcircled{\cap} m^{\mathcal{H}\uparrow \mathcal{X}\times\mathcal{H}}\right)^{\downarrow \mathcal{X}}$.

Marginalisation in the case of a product space (recalls)

- MF $m^{\mathcal{X}_1 \times \mathcal{X}_2}$ can be marginalised on $\mathcal{X}_1$ by transferring each mass $m^{\mathcal{X}_1 \times \mathcal{X}_2}(B)$, $B \subseteq \mathcal{X}_1 \times \mathcal{X}_2$, to the projection of B on $\mathcal{X}_1$:

$$m^{\mathcal{X}_1 \times \mathcal{X}_2 \downarrow \mathcal{X}_1}(A) = \sum_{\{B \subseteq \mathcal{X}_1 \times \mathcal{X}_2 \mid proj(B \downarrow \mathcal{X}_1) = A\}} m^{\mathcal{X}_1 \times \mathcal{X}_2}(B) \;, \forall A \subseteq \mathcal{X}_1 \;.$$

- Illustration



Vacuous extension in the case of a product space (recalls)

- Vacuous extension of MF $m^{\mathcal{X}_1}$ on $\mathcal{X}_1 \times \mathcal{X}_2$ is defined by (s-least committed solution, cf Lecture 2 of T. Denœux):

$$m^{\mathcal{X}_1 \uparrow \mathcal{X}_1 \times \mathcal{X}_2}(B) = \begin{cases} m_1^{\mathcal{X}}(A) & \text{if } B = A \times \mathcal{X}_2,\ A \subseteq \mathcal{X}_1 \\ 0 & \text{otherwise.} \end{cases}$$

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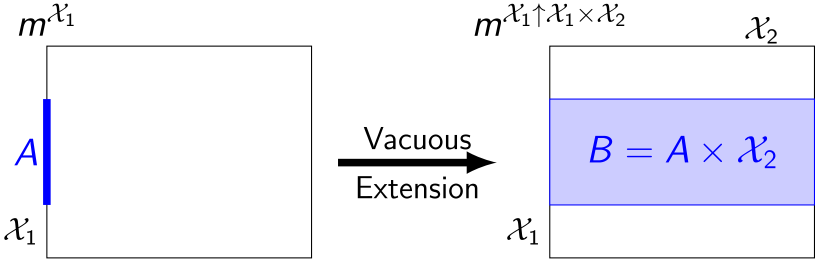

Conditioning in the case of a product space (recalls)

- With $D \subseteq \mathcal{X}_2$, the conditioning of a MF $m^{\mathcal{X}_1\times\mathcal{X}_2}$ is noted $m^{\mathcal{X}_1}[D]$ and defined by:

$$m^{\mathcal{X}_1}[D] = \left(m^{\mathcal{X}_1\times\mathcal{X}_2} \odot m_D^{\mathcal{X}_2\uparrow\mathcal{X}_1\times\mathcal{X}_2}\right)^{\downarrow\mathcal{X}_1} .$$

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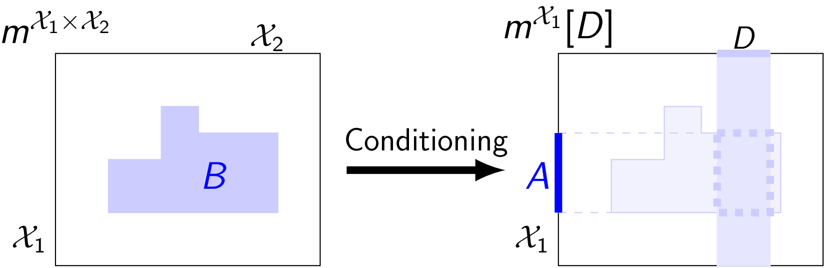

Deconditioning in the case of a product space (recalls)

- Deconditioning of a MF $m^{\mathcal{X}_1}[D]$ on $\mathcal{X}_1 \times \mathcal{X}_2$ is defined by (s-least committed solution):

$$m^{\mathcal{X}_1}[D]^{\Uparrow \mathcal{X}_1 \times \mathcal{X}_2}(A \times D \cup \mathcal{X}_1 \times \overline{D}) = m_1^{\mathcal{X}}[D](A), \quad \forall A \subseteq \mathcal{X}_1 \ .$$

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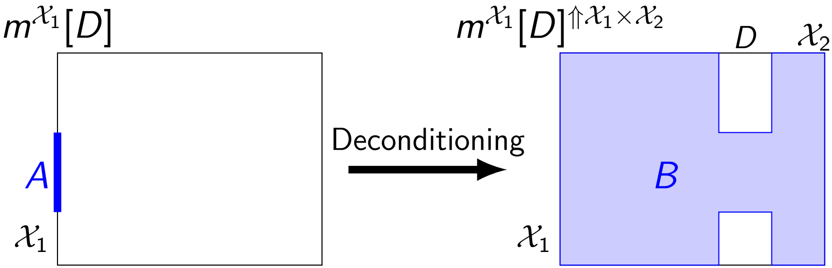

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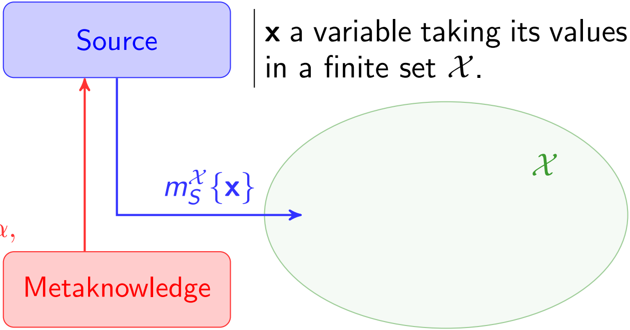

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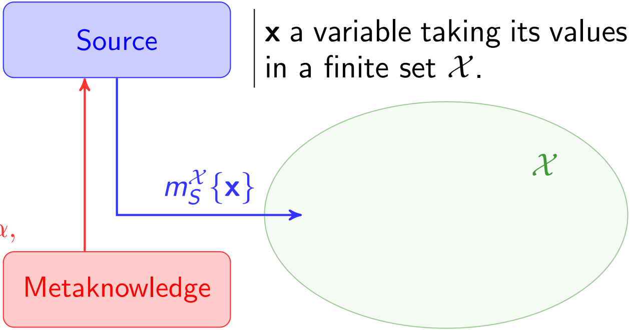

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Exercise: Do the computation (Solution in Smets 1993 Section 5.7 or Mercier et al. 2008 Section 2.5)

Discounting

Simpler expression

- Discounting of a MF m can be expressed, with $\alpha \in [0,1]$, by:

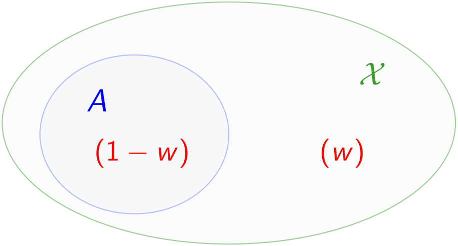
$$\begin{cases} {}^{\alpha}m(A) & = & (1-\alpha)m(A), \quad \forall A \subset \mathcal{X}, \\ {}^{\alpha}m(\mathcal{X}) & = & (1-\alpha)m(\mathcal{X}) + \alpha \ , \end{cases}$$

- or, using categorical (or logical) MF $m_{\mathcal{X}}$ ($m_{\mathcal{X}}(\mathcal{X}) = 1$):

$${}^{\alpha}m = (1-\alpha)m + \alpha m_{\mathcal{X}} \ .$$

Simple MF and negative simple MF (recalls)

- A MF m defined by $m(\mathcal{X}) = w$ and $m(A) = 1 - w$, with $w \in [0, 1]$ and $A \subset \mathcal{X}$, can be conveniently noted A^w. It is called a simple MF.



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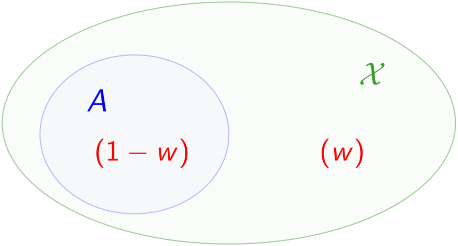

- A MF m such that $m(\emptyset) = v$ and $m(A) = 1 - v$, with $v \in [0, 1]$, $A \subseteq \mathcal{X}$, $A \neq \emptyset$, can be conveniently noted A_v. It is called a negative simple MF.

Negation of a MF (recalls)

- The negation $\overline{m}$ of a MF m is defined by $\overline{m}(A) = m(\overline{A})$ for all $A \subset \mathcal{X}$.
- we have:

$$\overline{A^w} = \overline{\left\{ \begin{array}{lll} \mathcal{X} & \mapsto & w \\ A & \mapsto & 1-w \end{array} \right.} = \left\{ \begin{array}{lll} \overline{\mathcal{X}} = \emptyset & \mapsto & w \\ \overline{A} & \mapsto & 1-w \end{array} \right. = \overline{A}_w \ .$$

Discounting
Expressions

- Discounting of a MF m can be expressed, with $\alpha \in [0,1]$, $\beta = 1-\alpha$, by:

$$\begin{cases} {}^{\alpha}m(A) &= \beta m(A), \quad \forall A \subset \mathcal{X}, \\ {}^{\alpha}m(\mathcal{X}) &= \beta m(\mathcal{X}) + \alpha \ , \end{cases}$$

- or, using categorical (or logical) MF $m_{\mathcal{X}}$ ($m_{\mathcal{X}}(\mathcal{X}) = 1$):

$${}^{\alpha}m = \beta m + \alpha m_{\mathcal{X}} \ ,$$

- or, using negative simple MF:

$${}^{\alpha}m = m \circledcup \mathcal{X}_{\beta} = m \circledcup \begin{cases} \emptyset & \mapsto \beta = 1-\alpha \\ \mathcal{X} & \mapsto \alpha \ . \end{cases}$$

Outline

Discounting

Contextual discounting based on a coarsening

Behaviour Based Correction (BBC)

Contextual discounting (CD), reinforcement (CR) and negating (CN)

Learning CD, CR and CN from labelled data

Contextual discounting based on a coarsening

Main idea (Mercier et al. 2005, 2008)

- The reliability of a source may depend on the true value of the variable of interest $\mathbf{x}$.

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- Example 1: A mercury thermometer reliable if the temperature is in the range $\{-38^{\circ}\text{C}, \ldots, 356^{\circ}\text{C}\}$

Contextual discounting based on a coarsening

Main idea (Mercier et al. 2005, 2008)

- The reliability of a source may depend on the true value of the variable of interest **x**.
- Example 1: A mercury thermometer reliable if the temperature is in the range $\{-38^\circ C, \ldots, 356^\circ C\}$
- Example 2: A cardiologist may be more reliable to diagnose cardiac disease than other kinds of disease.

- Let $\mathcal{A}$ be a partition of $\mathcal{X}$ representing different contexts
  - Example 1 (mercury thermometer): $\mathcal{A} = \{A_1, A_2\}$ with $A_1 = \{-38°\text{C},\ldots,356°\text{C}\}$ and $A_2 = \overline{A_1}$.

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- The source can be in two states: reliable (r) or not ($\neg r$), $\mathcal{H} = \{r, \neg r\}$, $m^{\mathcal{X}}[\{r\}] = m_S^{\mathcal{X}}$, $m^{\mathcal{X}}[\{\neg r\}] = m_{\mathcal{X}}$.

Contextual discounting based on a coarsening

Model

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- The source can be in two states: reliable (r) or not ($\neg r$), $\mathcal{H} = \{r, \neg r\}$, $m^{\mathcal{X}}[\{r\}] = m_S^{\mathcal{X}}$, $m^{\mathcal{X}}[\{\neg r\}] = m_{\mathcal{X}}$.
- For all contexts $A_k \in \mathcal{A}$: $m^{\mathcal{H}}[A_k](\{r\}) = \beta_{A_k}$ and $m^{\mathcal{H}}[A_k](\mathcal{H}) = \alpha_{A_k}$.
  - Example 1 (mercury thermometer): $\beta_{A_1} = 1$, $\beta_{A_2}=0$.

Contextual discounting based on a coarsening

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- The source can be in two states: reliable (r) or not ($\neg r$), $\mathcal{H} = \{r, \neg r\}$, $m^{\mathcal{X}}[\{r\}] = m_S^{\mathcal{X}}$, $m^{\mathcal{X}}[\{\neg r\}] = m_{\mathcal{X}}$.
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  - Example 1 (mercury thermometer): $\beta_{A_1} = 1$, $\beta_{A_2}=0$.
- If $\mathcal{A} = \{\mathcal{X}\}$ we retrieve the discounting model:
  - $m^{\mathcal{H}}[\mathcal{X}](\{r\}) = m^{\mathcal{H}}(\{r\}) = \beta$
  - $m^{\mathcal{H}}[\mathcal{X}](\mathcal{H}) = m^{\mathcal{H}}(\mathcal{H}) = \alpha$.

Contextual discounting based on a coarsening

Derivation

- Available evidence can be synthesized by:

$$\left(m^{\mathcal{X}}[\{r\}]^{\Uparrow \mathcal{X}\times\mathcal{H}} \bigcap_{A_k\in\mathcal{A}} m^{\mathcal{H}}[A_k]^{\Uparrow \mathcal{X}\times\mathcal{H}}\right)^{\downarrow \mathcal{X}}$$

Contextual discounting based on a coarsening

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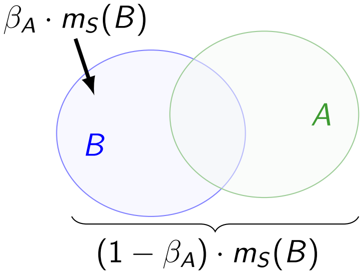
- Result is given by:

$$m_S \bigcirc\!\!\!\!\cup_{A \in \mathcal{A}} A_{\beta_A}$$

- Discounting is retrieved when $\mathcal{A} = \{\mathcal{X}\}$ ($\mathcal{A}$ contains one context $\mathcal{X}$):

$$m_S \bigcirc\!\!\!\!\cup \mathcal{X}_\beta$$

For each focal element B of m_S, for each context $A \in \mathcal{A}$:



- A portion $\beta_A \cdot m_S(B)$ remains on B.
- A portion $(1-\beta_A) \cdot m_S(B)$ is transferred to $B \cup A$.

Contextual discounting based on a coarsening

Example

With $\mathcal{X} = \{x_1, x_2, x_3\}$, $\mathcal{A} = \{\{x_1\}, \{x_2\}, \{x_3\}\}$, generalisation matrix associated with $m \circledcup \{x_1\}_{\beta_1} \circledcup \{x_2\}_{\beta_2} \circledcup \{x_3\}_{\beta_3}$ is given by:

$$
\begin{pmatrix}
\beta_1\beta_2\beta_3 & & & & & & & \\
\alpha_1\beta_2\beta_3 & \beta_2\beta_3 & & & & & & \\
\beta_1\alpha_2\beta_3 & & \beta_1\beta_3 & & & & & \\
\alpha_1\alpha_2\beta_3 & \alpha_2\beta_3 & \alpha_1\beta_3 & \beta_3 & & & & \\
\beta_1\beta_2\alpha_3 & & & & \beta_1\beta_2 & & & \\
\alpha_1\beta_2\alpha_3 & \beta_2\alpha_3 & & & \alpha_1\beta_2 & \beta_2 & & \\
\beta_1\alpha_2\alpha_3 & & \beta_1\alpha_3 & & \beta_1\alpha_2 & & \beta_1 & \\
\alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & \alpha_2 & \alpha_1 & 1
\end{pmatrix}
\quad
\begin{matrix}
\emptyset \\
\{x_1\} \\
\{x_2\} \\
\{x_1, x_2\} \\
\{x_3\} \\
\{x_1, x_3\} \\
\{x_2, x_3\} \\
\{x_1, x_2, x_3\}
\end{matrix}
$$

Outline

- A source provides a MF $m^{\mathcal{Y}}$,

Behaviour Based Correction (BBC)

Model (Pichon et al., 2012)

- A source provides a MF $m^{\mathcal{Y}}$,
- The state (or configuration) h in which stands the source, is described by a MF $m^{\mathcal{H}}$,

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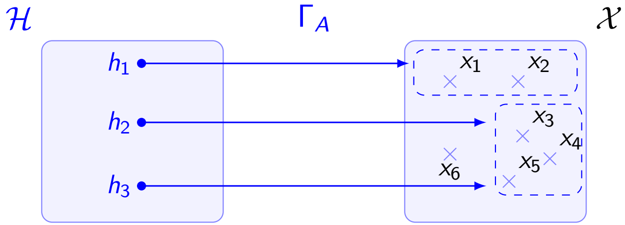
- A source provides a MF $m^{\mathcal{Y}}$,
- The state (or configuration) h in which stands the source, is described by a MF $m^{\mathcal{H}}$,
- For all $A \subseteq \mathcal{Y}$ a function Γ_A defined from $\mathcal{H}$ to $2^{\mathcal{X}}$ (Γ_A is a multi-valued mapping) indicates how to interpret the piece of information $y \in A \subseteq \mathcal{Y}$ for each state $h \in \mathcal{H}$.

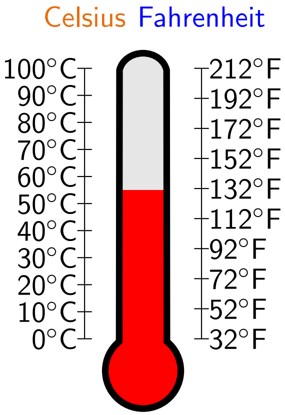


Behaviour Based Correction (BBC)

Examples

- Consider a thermometer that can be:
  - reliable: what it outputs is correct.
  - approx. (approximately reliable): if it provides a temperature t, the true one is in $\{t-1, t, t+1\}$.
  - unreliable: what it outputs is incorrect.
- Model:
  - $\mathcal{Y} = \mathcal{X}$ the set of temperatures,
  - $\mathcal{H} = \{\mathit{reliable}, \mathit{approx}, \mathit{unreliable}\}$,
  - and Γ_A is defined for all $A \subseteq \mathcal{X}$ by:

$$\begin{aligned}\Gamma_A(\mathit{reliable}) &= A\ ,\\ \Gamma_A(\mathit{approx}) &= A \cup_{t\in A} \{t-1, t+1\}\ ,\\ \Gamma_A(\mathit{unreliable}) &= \mathcal{X}\ .\end{aligned}$$



1. A source provides a MF $m^{\mathcal{Y}}$,
2. The state (or configuration) h in which stands the source, is described by a MF $m^{\mathcal{H}}$,
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Behaviour Based Correction (BBC)

Derivation

1. A source provides a MF $m^{\mathcal{Y}}$,
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- The 3rd piece of evidence defines a relation between $\mathcal{H}$, $\mathcal{Z} = 2^{\mathcal{Y}}$ and $\mathcal{X}$, which can be represented by the following logical MF:

$$m_{\Gamma}^{\mathcal{H}\times\mathcal{Z}\times\mathcal{X}}\left(\cup_{h\in\mathcal{H},z_A\in\mathcal{Z}}\{h\}\times z_A\times\Gamma_{z_A}(h)\right) = 1$$

with $\Gamma_{z_A}(h) = \Gamma_A(h)$ for all $h \in \mathcal{H}$ and $A \subseteq \mathcal{Y}$.

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  with $\Gamma_{z_A}(h) = \Gamma_A(h)$ for all $h \in \mathcal{H}$ and $A \subseteq \mathcal{Y}$.

- The 1st piece of evidence is a Bayesian MF s.t. $m^{\mathcal{Z}}(\{z_A\}) = m^{\mathcal{Y}}(A)$ for all $A \subseteq \mathcal{Y}$.

Behaviour Based Correction (BBC)

Derivation (continued) and Expression

- To have the information on $\mathcal{X}$, pieces of information $m^{\mathcal{Z}}, m^{\mathcal{H}}, \{\Gamma_{z_A}, A \subseteq \mathcal{Y}\} = m^{\mathcal{Z}}, m^{\mathcal{H}}, m_{\Gamma}^{\mathcal{H}\times\mathcal{Z}\times\mathcal{X}}$ are combined as follows

$$\left(\left(m^{\mathcal{Z}\uparrow\mathcal{H}\times\mathcal{Z}\times\mathcal{X}} \circledcap m_{\Gamma}^{\mathcal{H}\times\mathcal{Z}\times\mathcal{X}}\right)^{\downarrow\mathcal{H}\times\mathcal{X}} \circledcap m^{\mathcal{H}\uparrow\mathcal{H}\times\mathcal{X}}\right)^{\downarrow\mathcal{X}} .$$

Behaviour Based Correction (BBC)

Derivation (continued) and Expression

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- Result, called BBC and denoted by $f_{m^{\mathcal{H}}}(m^{\mathcal{Y}})$, is given by

$$f_{m^{\mathcal{H}}}(m^{\mathcal{Y}})(B) = \sum_{H\subseteq\mathcal{H}} m^{\mathcal{H}}(H) \sum_{A:\Gamma_A(H)=B} m^{\mathcal{Y}}(A) ,$$

for all $B \subseteq \mathcal{X}$, with $\Gamma_A(H) = \cup_{h\in H}\Gamma_A(h)$.

Behaviour Based Correction (BBC)

Derivation (continued) and Expression

- To have the information on $\mathcal{X}$, pieces of information $m^{\mathcal{Z}}, m^{\mathcal{H}}, \{\Gamma_{z_A}, A \subseteq \mathcal{Y}\} = m^{\mathcal{Z}}, m^{\mathcal{H}}, m_{\Gamma}^{\mathcal{H}\times\mathcal{Z}\times\mathcal{X}}$ are combined as follows

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for all $B \subseteq \mathcal{X}$, with $\Gamma_A(H) = \cup_{h\in H}\Gamma_A(h)$.

- *Exercise: Build the equivalent VBS (cf previous Lecture 9 of P. Shenoy) to derive the BBC.*

- Model:
  - $\mathcal{Y} = \mathcal{X}$, $m^{\mathcal{Y}} = m_S^{\mathcal{X}}$
  - $\mathcal{H} = \{\text{reliable}, \text{unreliable}\}$, s.t. Γ_A is defined $\forall A \subseteq \mathcal{X}$ by:

$$\begin{aligned} \Gamma_A(\text{reliable}) &= A \,, \\ \Gamma_A(\text{unreliable}) &= \mathcal{X} \,. \end{aligned}$$

  - $m^{\mathcal{H}}$ defined, with $\alpha \in [0,1]$, by:

$$\begin{aligned} m^{\mathcal{H}}(\{\text{reliable}\}) &= \beta = 1 - \alpha \,, \\ m^{\mathcal{H}}(\{\text{unreliable}\}) &= \alpha \,. \end{aligned}$$

- Gives: $f_{m^{\mathcal{H}}}(m_S^{\mathcal{X}}) = \beta m_S^{\mathcal{X}} + \alpha m_{\mathcal{X}}^{\mathcal{X}}$.

Behaviour Based Correction (BBC)

Example 2: Reinforcement of the mass of an element x_i of $\mathcal{X}$

- Model:
  - $\mathcal{Y} = \mathcal{X}$, $m^{\mathcal{Y}} = m_S^{\mathcal{X}} = m$
  - $\mathcal{H} = \{\text{reliable}, \text{reinf. of } x_i\}$, s.t. Γ_A is defined $\forall A \subseteq \mathcal{X}$ by:

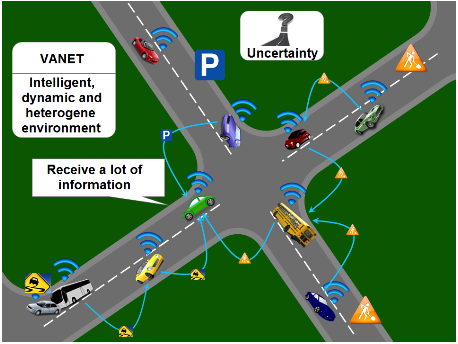
$$\begin{aligned}\Gamma_A(\text{reliable}) &= A \;, \\ \Gamma_A(\text{reinf. of } x_i) &= x_i \;.\end{aligned}$$

  - $m^{\mathcal{H}}$ defined, with $\alpha \in [0,1]$, by:

$$\begin{aligned}m^{\mathcal{H}}(\{\text{reliable}\}) &= \beta = 1 - \alpha \;, \\ m^{\mathcal{H}}(\{\text{reinf. of } x_i\}) &= \alpha \;.\end{aligned}$$

- Gives: $f_{m^{\mathcal{H}}}(m) = \beta m + \alpha m_{x_i}$, with $m_{x_i}(x_i) = 1$.

- Vehicles exchange messages about events happening on the road.
- Information about each event **e** is represented in each message by a MF $m^{\mathcal{X}}$ with $\mathcal{X} = \{\exists, \nexists\}$ and
  - $\exists$ meaning "event **e** exists",
  - $\nexists$ meaning "event **e** does not exist".



Two strategies for modelling messages ageings about accidents on the road:

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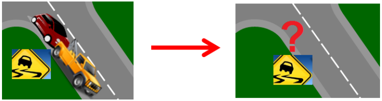
1. Either discount MFs $m^{\mathcal{X}}$: $(1-\alpha)m^{X} + \alpha m_{\mathcal{X}}$, with $\alpha \in [0,1]$ (over time, we do not know if the event is present or not).



Behaviour Based Correction (BBC)

Example 2 applied to VANETs (Bou Farah et al. 2016) 2/2

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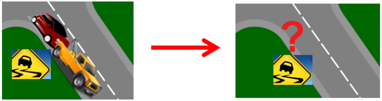
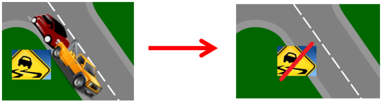


2. Or use the following mechanisms $(1-\alpha)m^X + \alpha m_{\{\nexists\}}$, with $\alpha \in [0,1]$ (over time we think the event is going to disappear).



Behaviour Based Correction (BBC)

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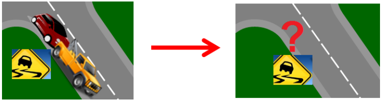

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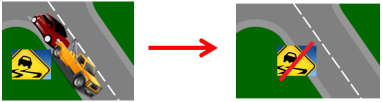

$\Rightarrow$ Experiments made show that the second strategy yields a better adequacy to the reality.

Outline

- Reliability is not limited to relevance.

- Reliability is not limited to relevance.
- Truthfulness: another dimension.
  - If a source is truthful, it gives the information it has.
  - If a source is not truthful (intentionally or not), it declares the contrary of what it knows. (crudest form)

CD, CR and CN

Example of a model with 2 dimensions

- $\mathcal{H} = \{(R, T), (R, \neg T), (\neg R, T), (\neg R, \neg T)\}$ with R meaning relevant and T truthful.

CD, CR and CN

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- $\mathcal{H} = \{(R, T), (R, \neg T), (\neg R, T), (\neg R, \neg T)\}$ with R meaning relevant and T truthful.
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$$\begin{aligned}
\Gamma_A(R, T) &= A\ , \\
\Gamma_A(R, \neg T) &= \overline{A}\ , \\
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\end{aligned}$$

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- $\mathcal{H} = \{(R,T),(R,\neg T),(\neg R,T),(\neg R,\neg T)\}$ with R meaning relevant and T truthful.
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- $m^{\mathcal{H}}$ defined, with $\alpha \in [0,1]$, $Prob(R) = p$ and $Prob(T) = q$, by:

$$\begin{array}{lcl} m^{\mathcal{H}}(\{R,T\}) & = & pq \\ m^{\mathcal{H}}(\{R,\neg T\}) & = & p(1-q) \\ m^{\mathcal{H}}(\{\neg R,T\}) & = & (1-p)q \\ m^{\mathcal{H}}(\{\neg R,\neg T\}) & = & (1-p)(1-q) \end{array}$$

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- BBC gives $f_{m^{\mathcal{H}}}(m_S^{\mathcal{X}}) = pq\, m_S^{\mathcal{X}} + p(1-q)\,\overline{m}_S^{\mathcal{X}} + (1-p)\, m_{\mathcal{X}}^{\mathcal{X}}$.

- $\neg T$ corresponds to the assumption that the source is non truthful for all values x_i of $\mathcal{X}$.

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- More subtle form of lack of truthfulness:
  - The source is non truthful for some values x_i of $\mathcal{X}$ and truthful for the other values of $\mathcal{X}$ (kind of contextual lack of truthfulness).

- $\neg T$ corresponds to the assumption that the source is non truthful for all values x_i of $\mathcal{X}$.
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- Examples:
  - State T corresponds to state $t_{\mathcal{X}}$.
  - State $\neg T$ corresponds to state $t_{\emptyset}$.

- Suppose
  - Source indicates $x \in B \subseteq \mathcal{X}$
  - Source is in state t_A (truthful for the values in A, untruthful for the values in $\overline{A}$)
- What can we conclude for x ?

CD, CR and CN

A first truthfulness refinement 2/2

- Suppose
  - Source indicates $x \in B \subseteq \mathcal{X}$
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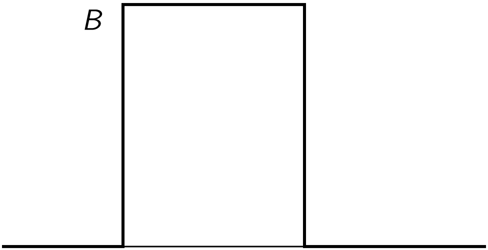
- What can we conclude for x ?



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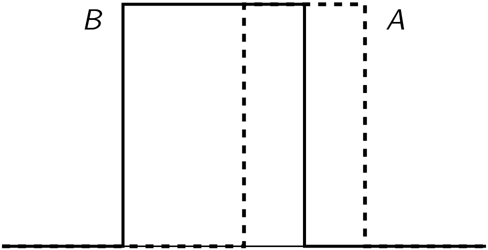

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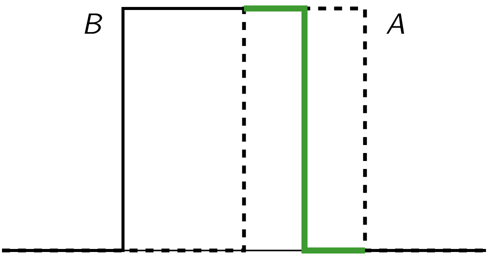

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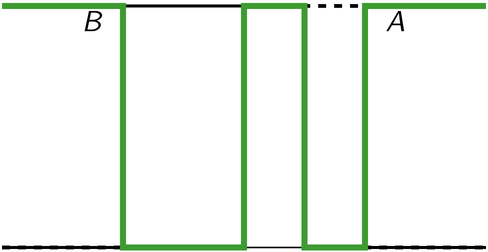

CD, CR and CN

A first truthfulness refinement 2/2

- Suppose
  - Source indicates $x \in B \subseteq \mathcal{X}$
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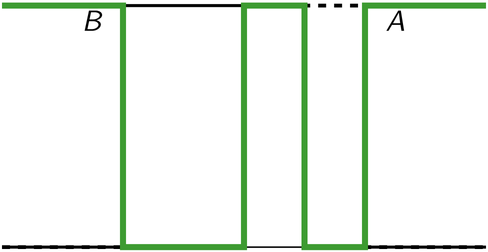
- What can we conclude for x ? $x \in (B \cap A) \cup (\overline{B} \cap \overline{A}) = B \underline{\cap} A$



CD, CR and CN

Contextual Negating: derivation from a composition of indep. BBCs (Pichon et al. 2016)

- With:
  - $\mathcal{H} = \{t_A | A \subseteq \mathcal{X}\}$ s.t. $\forall B \subseteq \mathcal{X}$ $\Gamma_B(t_A) = B \underline{\cap} A$ (in particular $\Gamma_B(t_\mathcal{X}) = B$)
  - A set $\mathcal{A}$ of contexts s.t. $\forall A \in \mathcal{A}$, MF $m^{\mathcal{H}}_{A,\underline{\cap}}$ is defined by:

$$\begin{array}{lcll} m^{\mathcal{H}}_{A,\underline{\cap}}(\{t_\mathcal{X}\}) & = & \beta_A & \text{(The source is truthful with a degree } \beta_A) \\ m^{\mathcal{H}}_{A,\underline{\cap}}(\{t_A\}) & = & \alpha_A & \text{(and untruthful in } \overline{A} \text{ with a degree } 1-\beta_A) \end{array}$$

where $\alpha_A \in [0,1]$, $\beta_A = 1 - \alpha_A$.

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where $\alpha_A \in [0,1]$, $\beta_A = 1 - \alpha_A$.

- We obtain (CN definition):

$$(\circ_{A\in\mathcal{A}} f_{m^{\mathcal{H}}_{A,\sqcap}})(m^{\mathcal{X}}_S) = m^{\mathcal{X}}_S \circledcap_{A\in\mathcal{A}} A^{\beta_A}$$
$$= m^{\mathcal{X}}_S \circledcap_{A\in\mathcal{A}} \begin{cases} \mathcal{X} \mapsto \beta_A \\ A \mapsto 1-\beta_A \end{cases}$$

CD, CR and CN

Contextual Negating: particular case of the negation of a MF

- With:
  - $\mathcal{A} = \{\emptyset\}$ (one context), denoted $\alpha_{\emptyset}$ simply by α, we have:

$$\begin{aligned} m^{\mathcal{H}}_{\emptyset,\sqcap}(\{s_{\mathcal{X}}\}) &= \beta \\ m^{\mathcal{H}}_{\emptyset,\sqcap}(\{s_{\emptyset}\}) &= \alpha \end{aligned}$$

  where $\Gamma_B(t_{\mathcal{X}}) = B$ (state $t_{\mathcal{X}}$ = truthful source) and $\Gamma_B(t_{\emptyset}) = \overline{B}$ (state $t_{\emptyset}$ = non truthful source).

- We have:

$$f_{m^{\mathcal{H}}_{\emptyset,\sqcap}}(m^{\mathcal{X}}_S) = \beta m^{\mathcal{X}}_S + \alpha \overline{m^{\mathcal{X}}_S}$$

  where $\overline{m^{\mathcal{X}}_S}(B) = m^{\mathcal{X}}_S(\overline{B})$, pour tout $B \subseteq \mathcal{X}$.

CD, CR and CN

2nd truthfulness refinement by considering positive and negative clauses 1/2

- Example of positive clause: x_i is a possible value for x.
- Example of negative clause: x_i is not a possible value for x.

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  - A source is said to be negatively truthful (resp. untruthful) for a value x_i of $\mathcal{X}$ if it declares that x_i is not a possible value for x and knows it is not (resp. it is).

CD, CR and CN

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- $\neg T$ corresponds to assuming that a source is positively and negatively non truthful for all values x_i of $\mathcal{X}$.
- It means two strong assumptions:
  1. The context (set of values) concerned by the lack of truthfulness is the entire frame $\mathcal{X}$.
  2. Both polarities are concerned by the lack of truthfulness

- This means we can consider states corresponding to weaker assumptions on the lack of truthfulness

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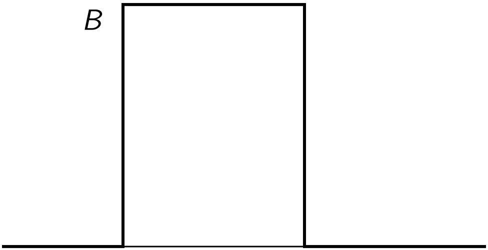
- Two are of particular interest:
  1. State p_A: Source truthful in A, negatively truthful and positively non truthful in $\overline{A}$.
  2. State n_A: Source is positively truthful and negatively non truthful in A, truthful in $\overline{A}$.

- Suppose
  - Source indicates $x \in B \subseteq \mathcal{X}$
  - Source is in state p_A (truthful in A, negatively truthful and positively non truthful in $\overline{A}$)
- What can we conclude for x ?



CD, CR and CN

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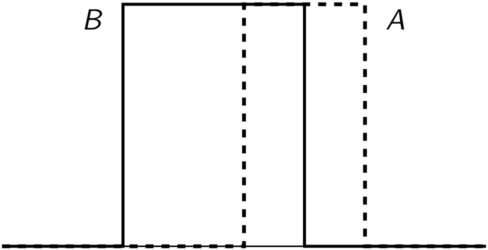

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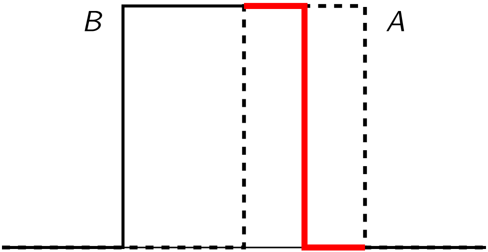

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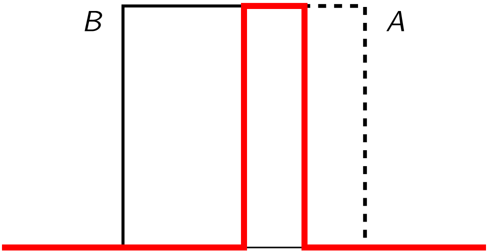

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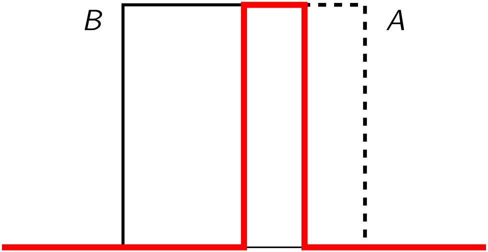

CD, CR and CN

Contextual Reinforcement: derivation from a composition of indep. BBCs (Pichon et al. 2016)

- With:
  - $\mathcal{H} = \{p_A | A \subseteq \mathcal{X}\}$ s.t. $\forall B \subseteq \mathcal{X}$ $\Gamma_B(p_A) = B \cap A$ (in particular $\Gamma_B(p_{\mathcal{X}}) = B$, state $p_{\mathcal{X}}$=truthful source)
  - A set $\mathcal{A}$ of contexts s.t. $\forall A \in \mathcal{A}$, MF $m^{\mathcal{H}}_{A,\cap}$ is defined by:

$$m^{\mathcal{H}}_{A,\cap}(\{p_{\mathcal{X}}\}) = \beta_A \quad \text{(The source is truthful with a degree } \beta_A\text{)}$$
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\end{array}
$$

  where $\alpha_A \in [0,1]$, $\beta_A = 1 - \alpha_A$.
- We obtain (CR definition):

$$
\begin{aligned}
(\circ_{A\in\mathcal{A}}\, f_{m^{\mathcal{H}}_{A,\cap}})(m^{\mathcal{X}}_S) &= m^{\mathcal{X}}_S \textcircled{\cap}_{A\in\mathcal{A}} A^{\beta_A} \\
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\end{aligned}
$$

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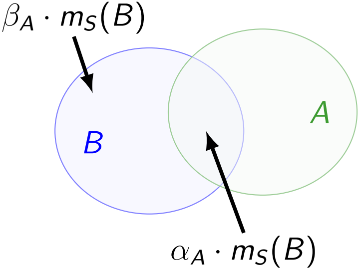
- For each focal element B of m_S, for each context $A \in \mathcal{A}$:

A part $\beta_A \cdot m_S(B)$ remains on B.

A part $\alpha_A \cdot m_S(B)$ is transferred to $B \cap A$.



CD, CR and CN

Contextual Reinforcement: an example

With $\mathcal{X} = \{x_1, x_2, x_3\}$, $\mathcal{A} = \{\{x_1\}\}$, specialisation matrix associated with $m \textcircled{\cap} \{x_1\}^{\beta_1} = m \textcircled{\cap} \left\{ \begin{array}{ccc} \mathcal{X} & \mapsto & \beta_1 \\ \{x_1\} & \mapsto & \alpha_1 \end{array} \right.$ is given by:

$$\begin{pmatrix} 1 & & \alpha_1 & & \alpha_1 & & \alpha_1 & \\ & 1 & & \alpha_1 & & \alpha_1 & & \alpha_1 \\ & & \beta_1 & & & & & \\ & & & \beta_1 & & & & \\ & & & & \beta_1 & & & \\ & & & & & \beta_1 & & \\ & & & & & & \beta_1 & \\ & & & & & & & \beta_1 \end{pmatrix} \quad \begin{array}{c} \emptyset \\ \{x_1\} \\ \{x_2\} \\ \{x_1, x_2\} \\ \{x_3\} \\ \{x_1, x_3\} \\ \{x_2, x_3\} \\ \{x_1, x_2, x_3\} \end{array}$$

CD, CR and CN

State n_A

- Suppose
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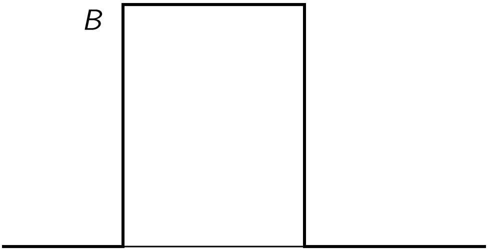

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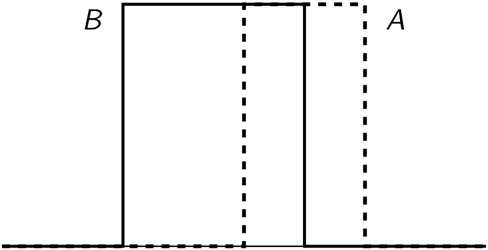

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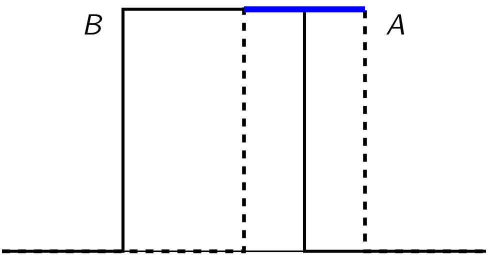

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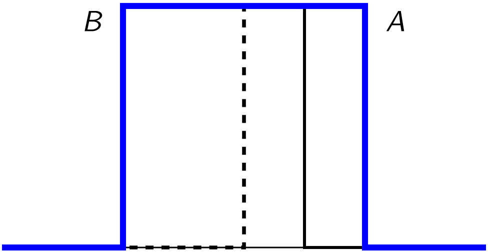

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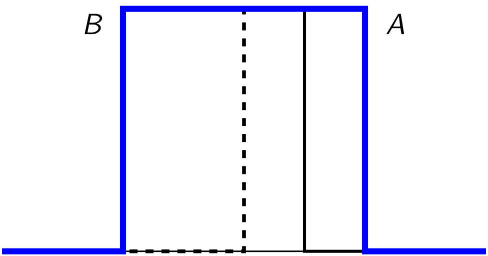

CD, CR and CN

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$$\begin{array}{lll} m^{\mathcal{H}}_{A,\cup}(\{n_\emptyset\}) & = & \beta_A \quad \text{(The source is truthful with a degree } \beta_A\text{)} \\ m^{\mathcal{H}}_{A,\cup}(\{n_A\}) & = & \alpha_A \quad \text{(and negatively untruthful in } A \text{ with} \\ & & \quad \text{a degree } 1-\beta_A\text{)} \end{array}$$

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$$

  where $\alpha_A \in [0,1]$, $\beta_A = 1 - \alpha_A$.
- We obtain(définition de CD) :

$$
\begin{aligned}
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&= m^{\mathcal{X}}_S \textcircled{\cup}_{A\in\mathcal{A}} \begin{cases} \emptyset \mapsto \beta_A \\ A \mapsto 1-\beta_A \end{cases}
\end{aligned}
$$

With $\mathcal{X} = \{x_1, x_2, x_3\}$, $\mathcal{A} = \{\{x_1\}\}$, generalization matrix associated with

$$m \textcircled{\cup} \{x_1\}_{\beta_1} = m \textcircled{\cup} \begin{cases} \emptyset & \mapsto & \beta_1 \\ \{x_1\} & \mapsto & \alpha_1 \end{cases}$$

is given by:

$$\begin{pmatrix} \beta_1 & & & & & & & \\ \alpha_1 & 1 & & & & & & \\ & & \beta_1 & & & & & \\ & & \alpha_1 & 1 & & & & \\ & & & & \beta_1 & & & \\ & & & & \alpha_1 & 1 & & \\ & & & & & & \beta_1 & \\ & & & & & & \alpha_1 & 1 \end{pmatrix} \begin{matrix} \emptyset \\ \{x_1\} \\ \{x_2\} \\ \{x_1, x_2\} \\ \{x_3\} \\ \{x_1, x_3\} \\ \{x_2, x_3\} \\ \{x_1, x_2, x_3\} \end{matrix}$$

Outline

Discounting

Contextual discounting based on a coarsening

Behaviour Based Correction (BBC)

Contextual discounting (CD), reinforcement (CR) and negating (CN)

Learning CD, CR and CN from labelled data

Learning CD, CR and CN from labelled data
Method

Labelled data:

1. n objects o_i, $i \in \{1, \ldots, n\}$ whose ground truth is known (classes belongs to $\mathcal{X} = \{x_1, \ldots, x_K\}$),
2. and the MF $m_S\{o_i\}$ output by S regarding the class of each object o_i,

Example with 4 objects (o_1, o_2, o_3 and o_4) and $\mathcal{X} = \{x_1, x_2, x_3\}$:

	$\{x_1\}$	$\{x_2\}$	$\{x_3\}$	$\{x_1, x_2\}$	$\{x_1, x_3\}$	$\{x_2, x_3\}$	$\mathcal{X}$	Truth
$m_S\{o_1\}$	0	0	0.5	0	0	0.3	0.2	a
$m_S\{o_2\}$	0	0.5	0.2	0	0	0	0.3	h
$m_S\{o_3\}$	0	0.4	0	0	0.6	0	0	a
$m_S\{o_4\}$	0	0	0	0	0.6	0.4	0	r

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$m_S\{o_2\}$	0	0.5	0.2	0	0	0	0.3	h
$m_S\{o_3\}$	0	0.4	0	0	0.6	0	0	a
$m_S\{o_4\}$	0	0	0	0	0.6	0.4	0	r

With the same idea as Zouhal and Denœux 1998, Elouedi et al. 2004, we can obtain the corrections (CD, CR and CN) of m_S by minimising a measure of discrepancy between the beliefs and ground truth.

Learning CD, CR and CN from labelled data

Chosen measure of discrepancy

- Chosen measure of discrepancy (between the corrected source output and the ground truth):

$$E_{pl}(\beta) = \sum_{i=1}^{n} \sum_{k=1}^{K} (pl\{o_i\}(\{x_k\}) - \delta_{i,k})^2$$

  - where $pl\{o_i\}$: plausibility function obtained from a contextual correction of m_S (CD, CR ou CN) with a parameter $\beta \in [0,1]^{|\mathcal{A}|}$.
  - and $\delta_{i,k} = 1$ if the class of o_i is x_k, $\delta_{i,k} = 0$ otherwise.

Learning CD, CR and CN from labelled data

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$$E_{pl}(\beta) = \sum_{i=1}^{n} \sum_{k=1}^{K} (pl\{o_i\}(\{x_k\}) - \delta_{i,k})^2$$

  - where $pl\{o_i\}$: plausibility function obtained from a contextual correction of m_S (CD, CR ou CN) with a parameter $\beta \in [0,1]^{|\mathcal{A}|}$.
  - and $\delta_{i,k} = 1$ if the class of o_i is x_k, $\delta_{i,k} = 0$ otherwise.
- Advantages of E_{pl} measure
  1. It yields to a least square optimization procedure (easy and quick to solve).
  2. It allows us to have an easy understanding of CD, CR and CN impacts on the measure.
  3. It is at least as justified as other measures.

CD :

- Minimum of $\mathbf{E_{pl}}$ is reached with $\boldsymbol{\beta} = (\beta_{\{x_k\}}, k \in \{1, \ldots, K\})$, which means with $\mathcal{A}$ composed of K contexts $\{x_k\}$, $k \in \{1, \ldots, K\}$.
- With this set of contexts $\mathcal{A}$, the plausibility on singletons after CD correction is defined for all $x \in \mathcal{X}$, with $\beta_{\{x\}} \in [0, 1]$, by:

$$pl(\{x\}) = 1 - (1 - pl_S(\{x\}))\beta_{\{x\}} \ .$$

- With $\beta_{\{x\}}$ varying in $[0, 1]$ one has for all $x \in \mathcal{X}$ (CD correction abilities):

$$pl(\{x\}) \in [pl_S(\{x\}), 1] \ .$$

CR :

- Minimum of $\mathbf{E_{pl}}$ is reached with $\boldsymbol{\beta} = (\beta_{\overline{\{x_k\}}}, k \in \{1, \ldots, K\})$, which means with $\mathcal{A}$ composed of K contexts $\overline{\{x_k\}}$, $k \in \{1, \ldots, K\}$.
- With this set of contexts $\mathcal{A}$, the plausibility on singletons after CR correction is defined for all $x \in \mathcal{X}$, with $\beta_{\overline{\{x\}}} \in [0, 1]$, by:

$$pl(\{x\}) = pl_S(\{x\})\beta_{\overline{\{x\}}} \ .$$

- With $\beta_{\overline{\{x\}}}$ varying in $[0, 1]$ one has for all $x \in \mathcal{X}$ (CR correction abilities):

$$pl(\{x\}) \in [0, pl_S(\{x\})] \ .$$

CN :

- Minimum of $\mathbf{E_{pl}}$ is reached with $\boldsymbol{\beta} = (\beta_{\overline{\{x_k\}}}, k \in \{1, \ldots, K\})$, which means with $\mathcal{A}$ composed of K contexts $\overline{\{x_k\}}$, $k \in \{1, \ldots, K\})$.
- With this set of contexts $\mathcal{A}$, the plausibility on singletons after CN correction is defined for all $x \in \mathcal{X}$, with $\beta_{\overline{\{x\}}} \in [0, 1]$, by:

$$pl(\{x\}) = 0.5 + (pl_S(\{x\}) - 0.5)(2\beta_{\overline{\{x\}}} - 1) \ .$$

- With $\beta_{\overline{\{x\}}}$ varying in $[0, 1]$ one has for all $x \in \mathcal{X}$ (CN correction abilities):

$$pl(\{x\}) \in [\min(pl_S(\{x\}), 1 - pl_S(\{x\})), \max(pl_S(\{x\}), 1 - pl_S(\{x\}))] \ .$$

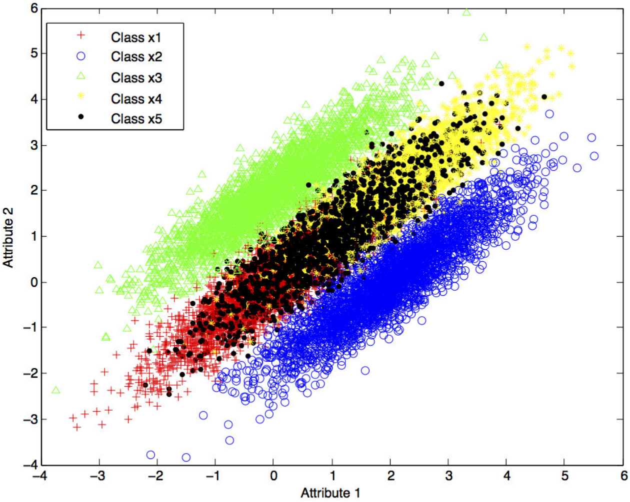
- **Goal**: we want to correct the information output by an evidential classifier using CD, CR and CN.
- The evidential k-nearest neighbour classifier (*ev-knn*) introduced by Denœux (1995) is chosen with $k = 3$.
- 5-class classification problem with data generated from 5 bi-variate normal distributions with respective means $\mu_{x_1} = (0,0)$, $\mu_{x_2} = (2,0)$, $\mu_{x_3} = (0,2)$, $\mu_{x_4} = (2,2)$, $\mu_{x_5} = (1,1)$ and common variance matrix

$$\Sigma = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}.$$

- **1000 instances** of each class are generated
- Total amount of data = **5000 instances**.

Learning CD, CR and CN from labelled data

An experiment in classification: Illustration of the 5000 instances

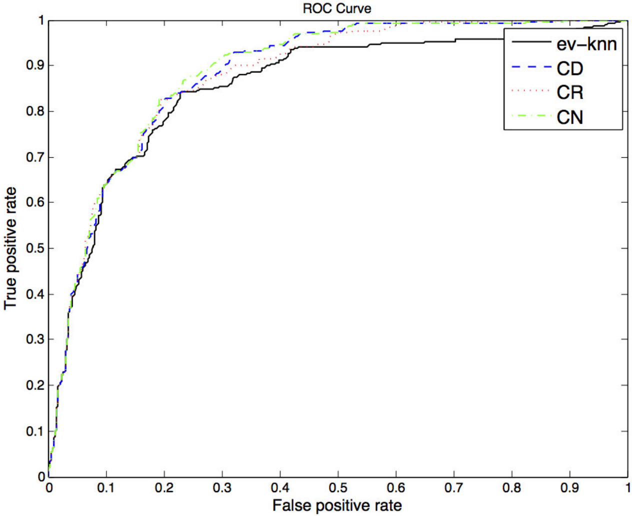


The 5000 instances are divided into 3 parts:

- 1/3 : Learning set for ev-knn.
- 1/3 : Learning set for CD, CR and CN.
- 1/3 : Test set.

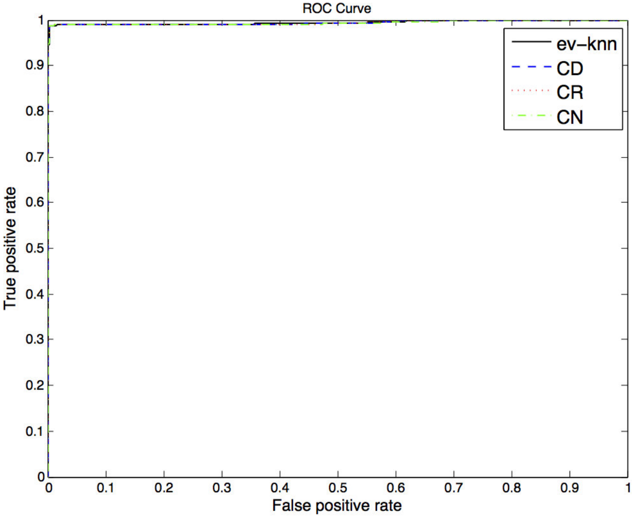
Learning CD, CR and CN from labelled data

An experiment in classification: Results for Class 1 ROC Curve



Learning CD, CR and CN from labelled data

An experiment in classification: Results for Class 2 ROC Curve



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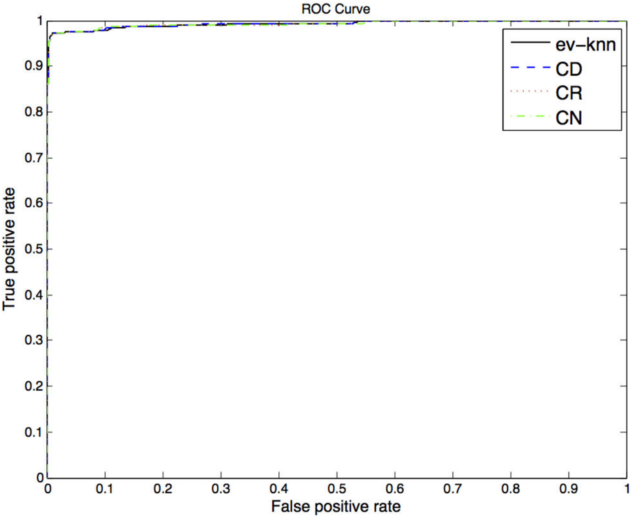
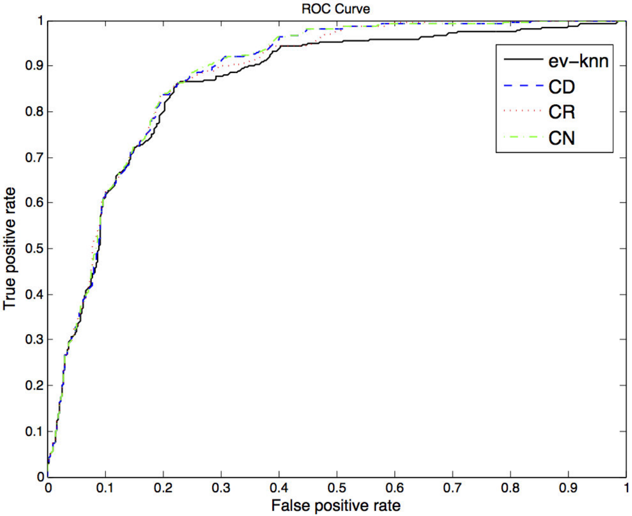
Learning CD, CR and CN from labelled data

An experiment in classification: Results for Class 3 ROC Curve



Learning CD, CR and CN from labelled data

An experiment in classification: Results for Class 4 ROC Curve



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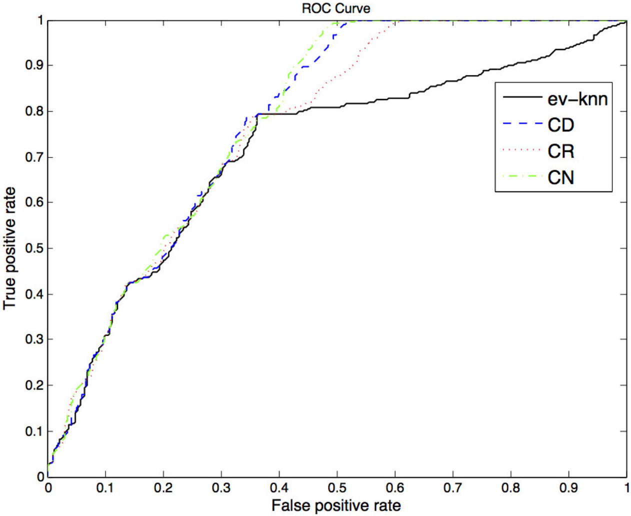
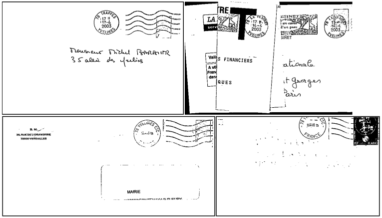
Learning CD, CR and CN from labelled data

An experiment in classification: Results for Class 5 ROC Curve



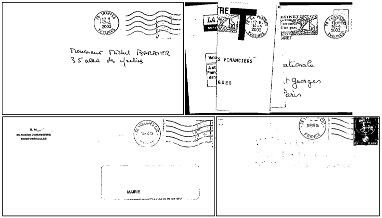
- An unknown classifier is available (black box) with maybe low or intermediate performances.
  - Example: a company buying sensors/classifiers from competitors (Mercier et al. 2009).



Learning CD, CR and CN from labelled data

Concluding remarks on the interest of the approach

- An unknown classifier is available (black box) with maybe low or intermediate performances.
  - Example: a company buying sensors/classifiers from competitors (Mercier et al. 2009).



- With these learning methods from labelled data, you can:

  1. improve the performances of this classifier;
  2. learn automatically its characteristics (Learnt parameters from the correction have an interpretation).

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Summary

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- Contextual corrections can be built in particular. They takes into account different possible behaviours of a source according to its outputs.
  - They can be automatically learnt from labelled data.
- Examples of applications with benefits from these corrections have been given.

What has not been presented

- The correction/adjustment of a group of sources.
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  - See Pichon et al. 2012 for consideration of joint state assumptions on sources.
  - See Mercier et al. 2008 for a learning from labelled data.
- Calibration of a source of information which provides a confidence score in addition to its output. See Xu et al. 2016, Minary et al. 2017

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Thank you for your attention.