Exchanging dynamic and imprecise information in V2V networks with belief functions

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Abstract—In this paper, the authors develop a system for exchanging and managing information about events on the road by considering in particular the spatial aspect of events such as traffic jam. Information on events being usually imperfect, belief functions are used to create messages representing the available information, to combine several pieces of information received and to give an overview of the situation to the driver. The proposed method has been implemented and tested.

I. INTRODUCTION

Ad-hoc networks are wireless networks capable to be organized without infrastructure. They are formed of wireless nodes communicating with each other to exchange information. In a context of mobility, they are called Mobile Ad-Hoc Networks (MANET) [1], [2], [3], [4].

In Vehicular Ad-Hoc Networks (VANET), nodes are highly dynamic. Two modes of communication are known about VANET applied to Inter-Vehicle Communication (IVC): Vehicle to Infrastructure (V2I) and Vehicle to Vehicle (V2V). The present work concerns V2V communication where vehicles do not use any centralized access point to build their own information assembly.

The environment is very proactive. Vehicles receive a large amount of information which is most of the time uncertain. Moreover, information coming from different sources could be related and describing the same event.

The theory of belief functions [5], [6] is a generalization of the probability theory. It constitutes a rich and flexible framework for representing and manipulating imprecise and non-certain information. It can offer a more satisfactory solution in some cases [7].

Hence the methods [8], [9] have been introduced to develop systems to share and manage imperfect information in V2V communication using the theory of belief functions, as well as the methods for detecting spiteful nodes [10] or information in front of vehicles [11], [12].

In this paper, the authors develop a system for exchanging and managing information about events on the road [9] by considering the spatial aspect of events such as traffic jam. To take into account this aspect, a notion of cell on a map depending on the type of the event is introduced. The method is tested using a developed MatlabTM simulator.

This paper is organized as follows. In Section II, needed basic concepts on belief functions are recalled. Then the proposed approach to tackle spatial events are exposed in

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Section III. The developed MatlabTM simulator as well as test experiments are presented in Section IV. Finally, Section V concludes and discusses future work.

II. BELIEF FUNCTIONS: BASIC CONCEPTS

In this paper, the Transferable Belief Model (TBM) is used to represent uncertainty and combine various pieces of information. Main notions of the TBM are given in this section. Readers can refer to [6] for a more complete description.

Two levels are distinguished in the TBM: the credal level where information are represented and manipulated by belief functions, and the pignistic or decision level where belief functions are transformed into probability measures to make decisions.

A. Representing Information

Let us consider a finite set $\Omega = \{\omega_1, \omega_2, ..., \omega_k\}$, called frame of discernment, of possible answers to a given question of interest. We do not know the true answer to the question, but we have information given by different sources regarding this answer. Each piece of information is represented by a basic belief assignment (BBA), also called a mass function m defined as a function from 2^{Ω} to [0,1], where 2^{Ω} is the set of all possible subsets of Ω , and such that the sum of all the masses is equal to $1: \sum_{A \subset \Omega} m(A) = 1$.

The quantity m(A) represents the part of the belief allocated to the fact that the true answer belongs to A, in particular $m(\Omega)$ represents the degree of ignorance of the source which has provided information m, and $m(\emptyset)$ represents the conflict [13], [14].

There exists a number of equivalent representations of m, for example the plausibility which is the maximum degree of belief that could be allocated to A. It is defined as $pl(A) = \sum_{B \cap A \neq \emptyset} m(B)$.

A belief mass can be assigned to a singleton or to a subset. The subsets A of Ω such that m(A)>0 are called the focal elements of m. When all focal elements are singletons, plausibility becomes a classical additive probability function. Belief functions are a generalization of probability functions since the size of focal elements can be greater than 1. In this case, plausibility becomes non-additive.

A belief mass allows the explicit modeling of conflict which form a fundamental difference with probability theory.

A mass function having less than two focal elements including Ω is called a *simple mass function*. A simple mass function m verifies $m(A)=1-\omega$ and $m(\Omega)=\omega$ with $A\subset\Omega$ and $\omega\in[0,1]$. It can be conveniently noted A^ω .

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B. Manipulating Information

1) Discounting: In order to consider the doubt regarding the reliability of the source providing the BBA, discounting procedure can be used to weak the mass function. The discounting operation [5, page 252] is defined by:

$$\begin{cases} {}^{\alpha}m(A) &= (1-\alpha)m(A), \quad \forall A \subset \Omega, \\ {}^{\alpha}m(\Omega) &= (1-\alpha)m(\Omega) + \alpha \end{cases}$$
 (1)

where $\alpha \in [0,1]$ is called the discount rate; coefficient $\beta = (1-\alpha)$ represents the degree of reliability regarding the information provided.

2) Conjunctive rule of combination: Suppose that two pieces of information quantified by m_1^{Ω} and m_2^{Ω} and expressed on Ω , are received from two distinct and reliable sources. These BBA can be combined using the conjunctive rule of combination [6], denoted \odot and defined by:

$$m_{1 \odot 2}(A) = \sum_{B \cap C = A} m_1(B) \, m_2(C) , \quad \forall A \subseteq \Omega .$$
 (2)

With this combination, masses are transferred to focal elements intersections.

C. Making a decision

In practice, the mass function m defined on Ω representing the available information regarding the answer to the question of interest, is obtained in credal level from a fusion process.

To make decisions in the TBM framework, a solution consists to transform the mass function m into a probability measure, and to choose the best hypothesis using the *pignistic probability* [15] distribution defined as:

$$BetP(\{\omega\}) = \sum_{\{A \subseteq \Omega, \omega \in A\}} \frac{m(A)}{|A| \ (1 - m(\emptyset))}, \quad \forall \omega \in \Omega. \ \ (3)$$

III. V2V PROCESS FOR EXCHANGING AND MANAGING IMPERFECT INFORMATION

In this section, a process is proposed to exchange and manage imperfect information in inter-vehicle communication. It allows vehicles to disseminate information with a certain confidence degree. This method extends the previous model introduced by the authors in [9], it considers not only non-spatial events such as accident, but also spatial events such as traffic-jam.

A. Map Representation

In the proposed model, in order to represent and manage information about events, traffic lanes are divided into small rectangular areas named *cells*, whose width is equal to the traffic lane width. An important consideration of the model exposed here concerns the fact that the length of the cells of a map depends on the type of the events considered (accident, traffic jam, etc.); it allows to save internal memory and bandwidth. Then information on each type of event is considered on different internal maps: one map for one type of event. For instance, information regarding parking spaces is considered with a more refined map than traffic-jam.

For each event type t, the cell identifier corresponding to a location ℓ on the map (a coordinate (x,y)) is given by $Cell(t,\ell)$. An example of a street with two traffic lanes divided into cells for roadworks area event type is illustrated in Figure 1.



Fig. 1: Example of a street representation. Cells in red are roadworks area.

B. Messages Representation and Creation

In order to exchange information about events on the road, vehicles create messages represented as a 5-tuple (S, t, d, ℓ, m) described in Table I.

TABLE I: Message Attributes

Attribute	Description
S	Source (vehicle) which has perceived the event
t	Type of the event (accident, roadworks, traffic jam,)
d	Date and Time when S has detected the event
ℓ	Location where S has detected the event
m	Mass function representing the confidence of S regarding
	the fact that the event is present

Throughout this paper, each attribute $x \in \{S, t, d, \ell, m\}$ of a message M will be denoted by M.x.

We emphasize that the source M.S of a message is not necessarily the source which has transferred the message M. Likewise date M.d is not the date of the reception of a message M, but the date of the creation of this message.

Attribute M.m denotes the mass function defined on the frame of discernment $\Omega = \{\exists, /\exists\}$, held by vehicle M.S, where \exists stands for "the event, which is of type M.t, is present at time M.d at location $M.\ell$.", and $\not\exists$ means "the event, which is of type M.t, is not present at time M.d at location $M.\ell$."

Drivers can broadcast information about perceived or nonperceived events on the road, by entering the following information (Figure 2):

- the type t of the event;
- the presence or not of the event, and the corresponding confidence $(M.m(\{\exists\}))$ if the vehicle perceives the event and $M.m(\{\not\exists\})$ otherwise);
- the event location with regard to the vehicle: same direction, opposite direction, on the right, or on the left.

Locations and dates are generated automatically using Global Positioning System (GPS).

C. Internal Database and Exchanged Messages

Each vehicle has an internal database regrouping created and received messages, where all messages $M^v_{e,i}$ concerning the same event e are grouped into a table M_e in the vehicle v database. An event e is a couple (t,c) where t is the type of the event and c is the cell where the event is located.

Vehicles exchange created and received messages present in their databases; they do not exchange fusion results.

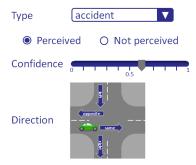


Fig. 2: Events entering interface

D. Message Reception Process

When a vehicle receives a message, the treatments are performed sequentially as illustrated in Figure 3.

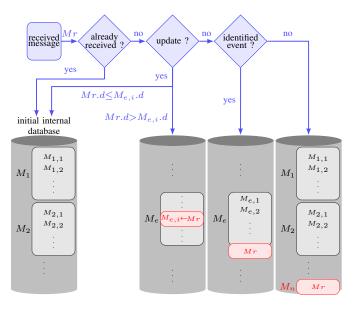


Fig. 3: Message Reception Treatment.

Details of these steps are given hereafter.

- 1) Message already received: If the received message M_r is already present in the vehicle database, it is not considered. Otherwise, we pass to the following step.
- 2) Received message is an update: The same source can broadcast a message, and repeat it later. A message M_1 is an update of a message M_2 if and only if $M_1.S = M_2.S$ (same source), $M_1.t = M_2.t$ (same type), $Cell(M_1.t, M_1.\ell) = Cell(M_2.t, M_2.\ell)$ (same cell) and $M_1.d > M_2.d$ (M_1 more recent). If a message $M_{e,i}^v$ is updated by M_r , it is replaced by M_r .

If a message $M_{e,i}^v$ present in the database is an update of the received message M_r , the latter is ignored.

3) Grouping messages corresponding to the same event: If M_r concerns an event already identified, it is added to the database. It is grouped in table M_e^v with e=(t,c), $M_r.t=t$ and $Cell(t,M_r.\ell)=c$. If M_r does not correspond to an event already identified, we pass to the last step.

4) Creating a new event: In this case, the received message M_r constitutes a new information on a new event: it is added to the vehicle database as a new event message (a new table M_e^v with $e = (M_r.t, Cell(M_r.t, M_r.\ell))$ is created).

E. Messages Obsolescence

In parallel to other processes, the system verifies if messages in vehicles databases are too old, in this case they are deleted. The deletion is realized using a threshold, denoted Del_t depending on the type t of the event: each message M such that $\Delta(now, M.d) > Del_{M.t}$ with Δ a distance, is suppressed.

F. Data Fusion: giving an overview of the situation to the driver

At this level, for each vehicle v and each event e, messages in table M_e^v are provided by distinct sources, and corresponding to the same event. An overview of the situation is given to drivers in the following manner:

- To take into consideration messages ageing, each mass function $M^v_{e,i}.m$ is discounted with a discount rate depending on the type t of the event and the date d of the message. It is defined by: $\alpha_{t,d} = \frac{\Delta(now,d)}{Delt}$.
- Second, for each event type t and each cell c occupied by t, discounted mass functions $\alpha^v_{e,i}M^v_{e,i}m$ are combined using the conjunctive rule of combination.
- At last, pignistic probabilities regarding each event presence are computed in each vehicle.

G. Consider the neighboring cells influences

The vehicle database can contain information about different parts of the road. In order to predict the overall road situation, a secondary mechanism is presented in this section. It permits to smooth result given to the driver concerning a spatial event zone.

The result of this mechanism is not communicated to other vehicles. It only allows to improve the overview of the situation given to driver.

The proposed mechanism can be explained in the following manner for each event type t:

- Let β_t be the influence rate.
- Step n°1: For each cell c occupied by t, generate influences on its neighborings (Figure 4) by discounting with a rate equal to $1 \beta_t$, and repeat this operation until the ignorance part of the resulting mass function tends to $1 \ (m(\Omega) > 0.99)$.

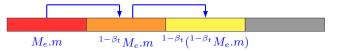


Fig. 4: Generation of influences on neighborings.

• Step n°2: The mass function corresponding to each cell c is then got by combining conjunctively obtained mass functions in the preceding step and mass functions in Section III-F.

IV. EXPERIMENTAL TESTS

A. Simulator

To illustrate the proposed method, a simulator has been developed in MatlabTM such that different scenarios can be tested. A scenario is made from the following information:

- for each type of event, a map composed of horizontal and vertical two-way streets as illustrated in Figure 5 (in particular a traffic lane is composed of NbSimCells_t cells depending on the type t of event);
- a scenario time axis τ with starting and end dates, and a sampling period Δτ;
- real events on the map denoted by R_i ;
- attributes related to vehicles such that the number of vehicles V, the network range NetRange in meter, the average vehicles speed in km/h when no event is present on the road, the slow-down rates, depending on event type, used to compute new vehicle speed when an event exists at its location at τ moment.

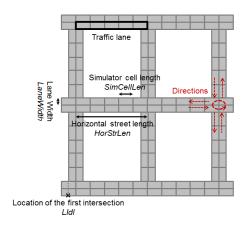


Fig. 5: Example of a map representation in the simulator.

At each τ moment, the following actions are realized:

- each vehicle creates a message when an event is present
 on its direction (in the cell where the vehicle is), on
 opposite direction (in the opposite cell), on the right
 (cell on the right at an intersection) or on the left (cell
 on the left at an intersection);
- each vehicle denies an event present in vehicle database if the driver does not perceive it;
- each vehicle communicates its messages to neighboring vehicles in the network range NetRange.

The method does not know the real size of the cells given by the simulator. Then for each type of event, the method gives information to drivers based on different cells whose size is computed from the number of cells per traffic lane denoted by $NbMethCells_t$.

B. Measure of Performance: Adequacy to the reality

The performance of the method is measured by considering the adequacy between the information given to the drivers in each vehicle and the reality.

The method giving information based on the fact that each traffic lane is composed of $NbMethCells_t$ cells, and the

reality given by simulator maps being based on the fact that each traffic lane is composed of $NbSimCells_t$ cells, the adequacy is assessed using the number of smallest common cells per traffic lane which is equal to the least common multiple of $NbMethCells_t$ and $NbSimCells_t$ denoted for short by $NbCCells_t$. A common cell is denoted by cc_t .

An example is illustrated in Figure 6.

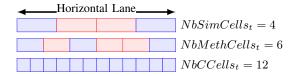


Fig. 6: Example of common cells computing for one type of event.

Performances can be computed on all the common cells of a map, or only on some of them, for example common cells present in a street.

Formally, performance measure of adequacy to reality concerning events of type t on a set C of common cells is given, for each vehicle v at each τ moment, by:

$$Perf_{t,C}^{\tau,v} = 1 - \frac{\sum_{cc \in C} \left(BetP_{t,cc}^{\tau,v}(\{\exists\}) - (t,cc)^{\tau}\right)^{2}}{|C|},$$
 (4)

where

- $(t, cc)^{\tau} = 1$ if an event of type t is present on cell cc at τ moment, 0 otherwise.
- $\mid C \mid$: number of common cells in C (cardinality of C).
- $BetP_{t,cc}^{\tau,v}$: pignistic probability of vehicle v at τ moment concerning the presence of an event of type t on the common cell cc. $BetP_{t,cc}^{\tau,v}(\{\exists\})=0$ if no event corresponding to (t,cc) is present in its database.

C. Experimentations

In this section, the method exposed in Section III is tested with different parameters in each scenario. The confidence of all created messages is equal to 0.6. Results in vehicles databases for each cell are illustrated by continuous succession of red, orange and yellow colors depending on pignistic probabilities: the higher is the pignistic probability $BetP(\exists)$, the closer is the cell color to red. The cells where no information is known by the vehicle are gray.

1) Scenario nº 1, spatial events: In this scenario, a vehicle is moving in two horizontal lanes where all the simulator cells contain spatial events traffic-jam as shown in Figure 9a.

This scenario lasts 2 minutes from 10:00:00 ($\tau=1$) to 10:02:00 ($\tau=22$). The number of simulator cells per traffic lane NbSimCells=4. For traffic-jam event type, influence rate $\beta=0.2$, and slow-down rate $slow-down\ rate=0.5$.

This scenario is tested with different values of the following parameters: number of method cells per traffic lane NbMethCells (3, 6, 12), time step $\Delta \tau$ (5, 10, 20) and vehicle speed (30km/h, 45km/h, 75km/h). When varying a parameter, following values of other parameters are used: NbMethCells = 6, $\Delta \tau = 10$, speed = 45km/h.

Results in vehicle database at the end of the simulation are shown in Figures 7 when varying each of these parameters. Blue squares correspond to τ moments (time steps).

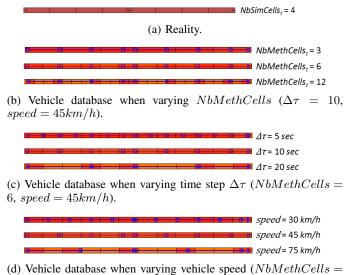


Fig. 7: Scenario nº1: reality and vehicle database.

In this spatial event zone scenario, the lesser is the method cells length, the lesser is the performance.

Time step and speed parameters affect the frequency at which vehicle does things (confirm or deny events, communicate messages, etc.). Therefore, an increase in the time step or the vehicle speed causes a decrease in performance.

At each time step, vehicle creates a new message confirming the event on a method cell. Without applying the influence mechanism, only these cells are occupied in vehicle database. The influence mechanism smoothes the obtained results by adding the event on neighboring cells.

Figure 8 shows the method performance without and with the influence secondary mechanism when the scenario parameters are the following: NbMethCells = 12, $\Delta \tau = 10$ and speed = 45km/h.

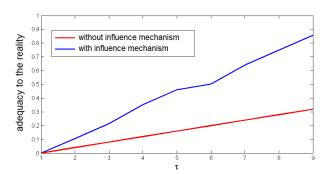


Fig. 8: Scenario n°1: method performance when $NbMethCells=12,~\Delta\tau=10$ and speed=45km/h.

In this scenario, the method performance holds on common cells concerned by the event in reality (common cells of the two simultaneous traffic lanes).

Without applying the influence mechanism, only cells where a message has been created are occupied in vehicle database. Performance on these cells is identical. It is equal to zero on the common cells occupied by the *traffic-jam* in reality and on which vehicle does not have information.

When applying the influence mechanism, neighboring cells (even those already occupied) inherit from cells where vehicle has created a message. For each cell, obtained mass functions are combined using the conjunctive rule of combination (Equation 2).

Performance on cells where vehicle has created a message is greater than performance on the other cells. It is due to the discounting of mass functions.

2) Scenario $n^o 2$, influence of cells length: In this scenario, an accident is present on a simulator cell. Accident being a non-spatial event, neighboring influence is not considered $(\beta = 0)$. Two vehicles are moving, each from a departure location to a destination location as shown in Figure 9a. Blue and green circled numbers correspond to τ moments.

This scenario lasts from 10:00:00 ($\tau=1$) to 10:00:44 ($\tau=22$). Time step $\Delta \tau$ is equal to 2 seconds. Average vehicles speed is equal to 45km/h when no event is present on the road, slow-down rate of accident event type is equal to 0.5. The number of simulator cells per traffic lane NbSimCells=4.

In order to show the influence of method cells length, this scenario is tested with four different numbers of method cells per traffic lane *NbMethCells*: 3, 4, 6 and 8. Results in vehicles databases at the end of the simulation are shown in Figures 9b and 9c.

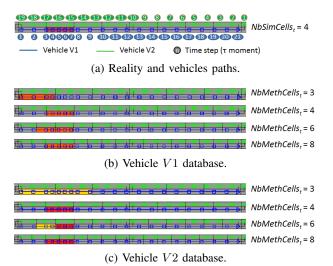


Fig. 9: Scenario n°2: reality and vehicles database results.

When NbMethCells=3, yellow cells in vehicle V2 database (Figure 9c) are denoted MC1 (cell located on the left) and MC2 (cell located on the right). In this case, the scenario unrolls as follows: at $\tau=3$, vehicles databases are empty, V1 creates a message to confirm the event on MC1 and updates it at $\tau=4$. From $\tau=5$ to $\tau=7$, V1 creates messages to confirm the event on MC2. At $\tau=8$ and $\tau=9$, V1 denies the event on MC2 (update):

then V1 thinks that no event is present on the method cell MC2. At $\tau=11$, V1 shares its database messages with V2. At $\tau=14$, V2 denies the event on MC2, then confirm the event (update) at $\tau=15$ and $\tau=16$. From $\tau=17$ to $\tau=19$, V2 denies the event on MC1. At the end of the simulation, V2 database contains conflicting messages concerning MC1 and MC2.

Let SC be the set of all simulator cells occupied by an event type. An incorrect or a conflicting result can be obtained in each method cell MCi where $MCi \notin SC$:

- Vehicle database may contain only information denying the existence of the event type on this cell. Example: the cell MC2 in V1 database when NbMethCells = 3.
- Vehicle database may contain messages confirming the existence of the event on this cell and others denying its existence. Example: yellow cells in V2 database when NbMethCells = 3 and NbMethCells = 6.

Experiments show how the proposed method manages spatial events. On the other hand, results depend on method cells length and influence mechanism rate β_t .

The automatic computation of the method cells sizes and the values of β_t are still under development; currently they are set manually. A possible solution consists in using historical knowledge to study these parameters; each vehicle can build its own knowledge, and can share it with other vehicles.

The method cells sizes can also be studied by choosing a small method cell length and have a short time step if bandwidth and databases spaces permit. For example, we can suppose that *traffic-jam* takes place starting from ten successive vehicles driving very slowly; then the method cell length is equal to ten times the average length of a vehicle.

V. CONCLUSIONS AND FUTURE WORK

In this paper, a method is proposed to exchange and manage information about events on the road in V2V communication taking into account non spatial and spatial events. Authors propose to divide map traffic lanes into small rectangular areas named *cells*.

Vehicles create and communicate messages in order to confirm, update or deny events. The proposed method uses belief functions to combine this information. A secondary mechanism allows smoothing results by considering neighboring influence, it improves the method performance. In order to help drivers making decisions, the method gives them an overview of the road situation. Implemented tests illustrate the interest of this method.

A first method has been implemented as a smartphone application in order to be used by drivers, back-seat drivers or even by pedestrians to share other types of events such as bag snatcher or long entrance queue [9]. This application must be upgraded with the presented method and tested in real situation. Sensors might be used to detect events in order to create messages automatically, without driver assistance.

In prospects, irregular areas and other types of spatial events such as flog blanket (where opposite traffic lane cells are influenced) must be considered.

In the presented method, location provided by drivers could be on the borderline between two cells. In future work, it can be interesting to consider the inaccuracies of location.

When vehicles have some information about events on the road, each driver generally reacts and changes course to reach his/her destination. Vehicles reactions can then to be considered in a new simulator.

Another point concerns the links between different types of event. An event can generate another event, e.g. an accident can generate a traffic jam. Future studies will then be devoted to the treatments of these elements.

VI. ACKNOWLEDGMENTS

This work is financed by the French region Nord-Pas de Calais under the project CISIT (Campus International pour la Sécurité et l'Intermodalité des Transports).

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